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An experimental and analytical study of instability of asymmetric jetstream-like currents in a rotating fluid

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Considering the barotropic instability problem of the mean westerly current in the atmosphere we have performed a series of experiments in a rotating vessel (using water and a barotropic model) to study the behaviour of a zonal asymmetric basic current with respect to small perturbations. In the centre of a rotating cylindrical vessel (of large diameter and rotation rate ω) a smaller cylinder was installed, the rotation of which relative to the vessel, at a rate $\Delta\omega$, generates a nearly two-dimensional field of mean relative motion within a sharply defined region. The dominant zonal velocity component $\bar{v}$ shows monotonic radial decrease within this so-called friction zone. Now what happens if the relative rotation of the inner cylinder, the source of momentum, suddenly vanishes, i.e. $\Delta\omega = 0$? The main result is that the basic zonal current $\bar{v}$, which now has an asymmetric radial profile ($\bar{v} = 0$ at the inner cylinder and the outer edge of the friction zone), breaks down into vortices, the number of which, the integer wavenumber n, is a function of the parameter $\epsilon = \Delta\omega/\omega$ alone: $n = n(\epsilon)$; increasing ϵ effects a decrease of n. For a theoretical discussion of the experimental results we assume this to be a problem of barotropic instability and base our analytical considerations on the two-dimensional non-divergent vorticity equation, frictional forces being neglected. By applying a perturbation method and prescribing a realistic asymmetric basic current we can derive the relation $\nu = \{[\frac{3}{4}\pi/\ln(\epsilon+1)^{\frac{1}{2}}]^2+1\}^{\frac{1}{2}}$, which yields the real wavenumber ν as a function of the parameter $\epsilon = \Delta\omega/\omega$. The analytical results are in good agreement with the experiments.

1. Introduction

Recently, in the course of an investigation of angular momentum exchange in a rotating fluid (Dunst 1972) the steady-state phenomenon of the so-called 'friction zone' has been studied in some detail. This friction zone produced by a concentrically rotating inner cylinder acting as a source of momentum can be characterized in the following way. (i) It is restricted to a narrow area around the inner cylinder; only in this area do we find a field of relative motion. (ii) Essentially its radial extent b is a function of the two parameters R and ϵ: $b = F(R, \epsilon)$, where R is the radius of the inner cylinder and $\epsilon = \Delta\omega/\omega = (n_2-n_1)/n_1$ (n_1 and n_2 are the rotation rates in revolutions per minute of the vessel and the inner cylinder, respectively). In the case $\epsilon > 0$ a simple formula for b which is in good agreement with the experimental results can be derived: $b = R[(\epsilon+1)^{\frac{1}{2}} - 1]$. (iii) The field of mean zonal motion within the friction zone is nearly two-dimensional; the dominant zonal velocity component $\bar{v}$ decreases monotonically from the inner cylinder to the outer edge of the friction zone.

Now the obvious question arises as to what will happen if after producing a friction zone the relative rotation of the inner cylinder is stopped suddenly ($\Delta\omega = \frac{2}{60}\pi\,(n_2 - n_1) = 0$), i.e. the source of momentum vanishes. For a short time after $\Delta\omega$ is set to zero the mean nearly barotropic zonal velocity component $\bar{v}$, which is now zero at the inner cylinder, reaches its maximum value a short distance off the cylinder before decreasing again towards zero at the outer boundary. This asymmetric radial profile for $\bar{v}$ resembles to a fair degree certain jetstream-like flows and the proper question now to be answered is how such mean currents will behave with respect to small perturbations; whether or not, for example, they will break down into vortices.

The experimental and analytical approach to this problem will be confined to the cases where $\epsilon > 0$, because then the friction zone, which serves as the initial state of our problem, can best be realized experimentally and be represented through a simple analytical formula.

This investigation is connected with the problem of the barotropic stability of the mean westerly current in the upper atmosphere (Kuo 1949; Long 1960; Lipps 1962) because in either case the basic current possesses a non-uniform lateral profile.

2. The experiments

For the experiments the following model arrangement was used (see figure 1). A cylindrical Plexiglas vessel Z_a (diameter 100 cm, height 50 cm), filled with water to a height of 20 cm (with a free surface), rotates around a vertical axis with angular velocity $\omega\,\mathrm{s}^{-1}$, where ω satisfies the relation $\omega = \frac{2}{60}\pi n_1$, n_1 being in revolutions per minute; in our experiments $0 < n_1 < 50$. In the centre of Z_a a smaller inner Plexiglas cylinder Z_i is installed, and can rotate independently around the same axis with angular velocity $\omega' = \frac{2}{60}\pi n_2$; $0 < n_2 < 80$ r.p.m. The rotation rate of Z_i relative to the vessel (system of reference) is given by $\Delta\omega = \omega' - \omega = \frac{2}{60}\pi(n_2 - n_1)$. The flow in the vessel is made visible using different coloured agents, which diffuse into the water through a very small slit (0·2 cm wide) on the bottom between the inner cylinder and the wall of the vessel. The field of relative motion can be photographed from above by a camera attached to the rotating vessel.

As the initial state in our experiments we consider a friction zone (steady state) of limited radial extent, which is produced by a certain relative rotation of the inner cylinder, the essential parameter being the ratio $\epsilon = \Delta\omega/\omega = (n_2 - n_1)/n_1$. In the case $\epsilon > 0$, to which our experiments are confined, the radial extent b of the friction area can be calculated by the formula, already mentioned above,

$$b = R[(\epsilon+1)^{\frac{1}{2}} - 1], \tag{1}$$

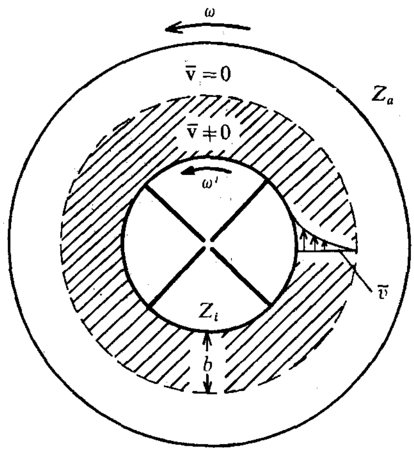

FIGURE 1. View of experimental arrangement in the initial state. Z_a is a cylindrical Plexiglas vessel with angular velocity ω (diameter 100 cm), Z_i the inner Plexiglas cylinder with angular velocity ω'; b the radial extent of the friction zone, $\bar{\mathbf{v}}$ the mean relative motion and $\bar{v}$ the zonal component of $\bar{\mathbf{v}}$, having a monotonic radial profile.

where R is the radius of the inner cylinder. Moreover, both the turbulent character of the initial state and the monotonic radial decrease of $\bar{v}(r)$ (the zonal component of the relative motion $\bar{\mathbf{v}}$) depend upon ϵ. The turbulent intensity grows with increasing values of ϵ, whereas $\partial\bar{v}/\partial r$ is reduced. $\epsilon > 0$ means that the inner cylinder rotates in the same direction as the vessel but faster. In analogy with meteorological practice, the counter-clockwise rotation of the vessel is called cyclonic; then for $\epsilon > 0$ we have anticyclonic shearing vorticity: $\partial\bar{v}/\partial r < 0$. In figure 1, the shaded area schematically represents the friction zone with the radial profile of $\bar{v}$.

It should be mentioned here that in addition to ϵ the water temperature, which affects the kinematic viscosity, i.e. the energy dissipation, may also have some influence on the turbulent intensity of the initial state. This may be important for $0 < \epsilon < 1$, where only a relatively small amount of turbulent energy is available. To eliminate such possible temperature effects all experiments have been performed at a constant water temperature of 18 °C.

Now, a typical experiment runs as follows. After a certain friction zone has been generated, the source of momentum is suddenly removed by setting $\Delta\omega = \frac{2}{60}\pi\,(n_2 - n_1) = 0$. Only a few seconds later wavelike disturbances of the zonal flow occur, and grow rapidly, until the whole flow falls into several vortices, which by and by will lose their energy by direct dissipation or disintegrate into even smaller vortices.

The following six experiments, the data for which are listed in table 1, have been chosen as characteristic examples for the phenomenon. By using some coloured dye and strewing the surface with very small plates (0·08 cm diameter) the individual phases of the experiments could be viewed and photographed from above, the coloured regions being reproduced by dark tones; see figures 2–7 (plates 1–6).

Experiment	Initial state ϵ	n_1	n_2	Radius of Z_i, R (cm)	Water temperature (°C)	Figures
1	0·33	30	40	7·5	18	2 (*a*)–(*c*) (plate 1); time after $\Delta\omega = 0$: (*a*) 50 s, (*b*) 80 s, (*c*) 110 s
2	0·66	30	50	7·5	18	3 (*a*)–(*c*) (plate 2); time after $\Delta\omega = 0$: (*a*) 15 s, (*b*) 30 s, (*c*) 45 s
3	1·0	25	50	5·0	18	4 (*a*)–(*c*) (plate 3); time after $\Delta\omega = 0$: (*a*) 20 s, (*b*) 35 s, (*c*) 55 s
4	3·0	10	40	7·5	18	5 (*a*)–(*c*) (plate 4); time after $\Delta\omega = 0$: (*a*) 15 s, (*b*) 20 s, (*c*) 25 s
5	5·0	5	30	5·0	18	6 (*a*)–(*c*) (plate 5); time after $\Delta\omega = 0$: (*a*) 25 s, (*b*) 35 s, (*c*) 50 s
6	16·0	3	51	5·0	18	7 (*a*)–(*c*) (plate 6); time after $\Delta\omega = 0$: (*a*) 15 s, (*b*) 40 s, (*c*) 55 s

TABLE 1. Data for the six experimental examples

3. Experimental results

Basic current

For a short time after setting $\Delta\omega = 0$ the zonal velocity component $\bar{v}$ shows a non-uniform radial distribution, which is illustrated schematically by figure 8. At the inner cylinder $\bar{v}$ vanishes, whereas only a short distance off the cylinder $\bar{v}$ will reach its maximum value before decreasing again monotonically towards zero at the outer boundary of the friction zone. This asymmetric radial profile of $\bar{v}$, similar to certain jetstream-like velocity profiles in the atmosphere, depends upon ϵ in a characteristic way, such that increasing ϵ results in an increasing asymmetry. Unfortunately we were not able to verify the exact form of the profile $\bar{v}\ (r, \epsilon)$ by direct measurements.

Decay of basic motion

The main experimental result is that the basic motion breaks down into vortices, the number of which, the wavenumber n, is a function of ϵ alone, in that increasing ϵ effects a decrease of n. As our experiments have proved, there is no dependence on the radius R of Z_i, for R varying in the range $5 \leqslant R \leqslant 15$ cm.

In figure 9 the observed integer wavenumbers n are plotted against the corresponding values of ϵ. Each experiment was repeated six times. Solid circles represent those experiments which yielded the same wavenumber six times (for the same ϵ). Open circles denote the cases where the same n was obtained only four times; in the remaining two repetitions another wavenumber was found for the same ϵ; these are marked by triangles.

It should be added here that in a more precise sense each of the vortices referred to is a pair consisting of a strong and almost circular cyclonic vortex and a smaller, weak and distorted anticyclonic vortex. But this structure can only be identified for the range $2 < \epsilon < 14$; for smaller and greater values only cyclonic

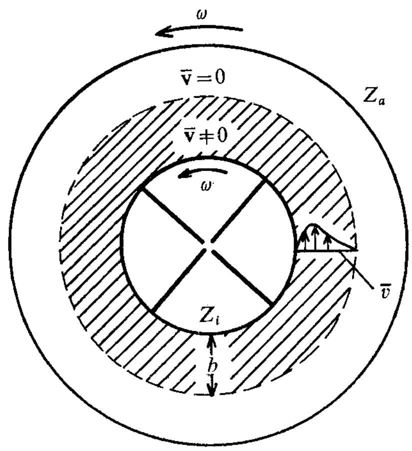

FIGURE 8. View of experimental configuration in the non-steady state after $\Delta\omega = 0$ (at the first moment). Z_a, Z_i, b and $\bar{\mathbf{v}}$ are as in figure 1 ($\omega' = \omega$). $\bar{v}$, the zonal component of $\bar{\mathbf{v}}$, has a jetstream-like radial profile.

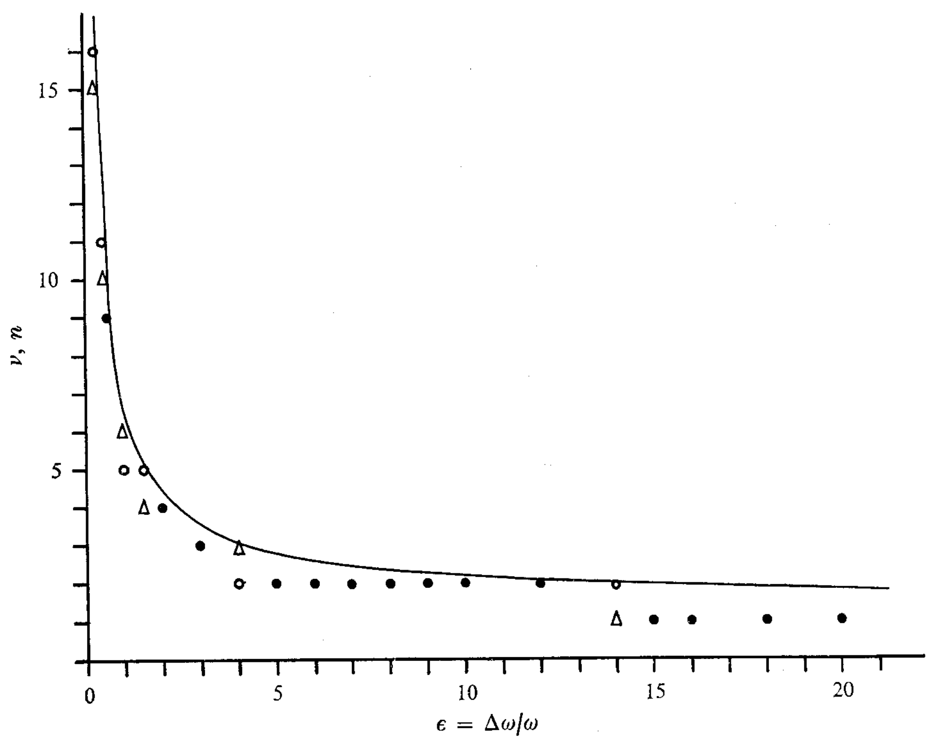

FIGURE 9. Wavenumbers n (observed) and ν (theoretical) as functions of ϵ; —, theoretical result (according to relation (23)). Experimental results: ●, repeated 6 times for same ϵ; ○, 4 times; △, twice.

vortices can be observed. For $0 < \epsilon < 1$ the small cyclonic vortices remain near the inner cylinder, their rotational energy being gradually dissipated. For greater values of ϵ the vortices drift into the outer region of the vessel (see figures 6 and 7, plates 5 and 6).

As our experiments have revealed, we always find the wavenumber 2 (two vortices) in the range $5 < \epsilon < 15$ (see figure 7); but with increasing ϵ within this interval the radial extents of the two cyclonic vortices become more and more different, being approximately equal for $\epsilon = 5$ (see figure 6), until we observe only one large vortex for $\epsilon = 15$ (or 16) (see figure 7).

4. Theoretical discussion

Definition of the problem

For a theoretical treatment of the phenomena described above we assume this to be a problem of barotropic instability. An asymmetric basic current $\bar{v}(r, \epsilon)$, which is allowed to vary in radial direction only and has to satisfy certain boundary conditions, is affected by small perturbations. Now the question is how this current will react to these perturbations. Frictional forces will be neglected.

Our considerations are based on the two-dimensional non-divergent vorticity equation, which describes our barotropic motion. This equation states that for any fluid element the vertical component of absolute vorticity is conserved. The absolute vorticity includes both the relative vorticity due to motion relative to the vessel and the constant vorticity 2ω of the vessel's rotation.

In cylindrical co-ordinates (r, ϕ) (see figure 10) the vorticity equation and the equation of continuity take the forms

$$\frac{\partial}{\partial t}(r\eta) + (ru)\frac{\partial \eta}{\partial r} + v\frac{\partial \eta}{\partial \phi} = 0, \tag{2}$$

$$\partial(ru)/\partial r + \partial v/\partial \phi = 0, \tag{3}$$

where $\eta = r^{-1}(\partial(rv)/\partial r - \partial u/\partial \phi)$ denotes the relative vorticity, u the radial and v the zonal velocity component relative to the vessel.

The perturbation equations

Let us consider a motion slightly disturbed from a pure zonal basic current:

$$v = \bar{v}(r, \epsilon) + v'(r, \phi, t), \quad u = u'(r, \phi, t). \tag{4}$$

Then the perturbations u', v' and η' satisfy the linearized equations

$$\frac{\partial}{\partial t}(r\eta') + (ru')\frac{d\bar{\eta}}{dr} + \bar{v}\frac{\partial \eta'}{\partial \phi} = 0, \tag{5}$$

$$\partial(ru')/\partial r + \partial v'/\partial \phi = 0, \tag{6}$$

with $\bar{\eta} = \dfrac{1}{r}\dfrac{d}{dr}(r\bar{v}), \quad \eta' = \dfrac{1}{r}\left(\dfrac{\partial}{\partial r}(rv') - \dfrac{\partial u'}{\partial \phi}\right), \quad R \leqslant r \leqslant R + b = R(\epsilon + 1)^{\frac{1}{2}},$

if (1) is used for the radial extent b of our initial field of motion.

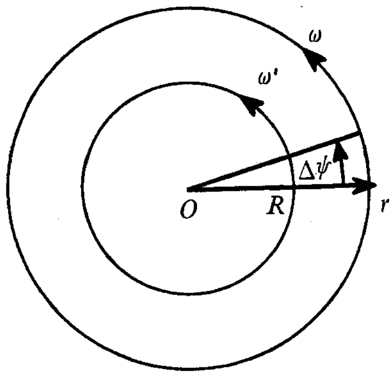

FIGURE 10. Cylindrical co-ordinates (r, ϕ). ω and ω' are the angular velocities of the vessel and the inner cylinder around the vertical axis. R is the radius of the inner cylinder.

Now it is possible to set up an 'equivalent' plane motion by defining the following dimensionless quantities:

$$t_* = tv_0/R, \quad x = \phi, \quad y = \ln(r/R), \quad 0 \leqslant y \leqslant \ln(\epsilon+1)^{\frac{1}{2}},$$

$$\overline{w} = \frac{\overline{v}}{r}\frac{R}{v_0}, \quad u'_* = \frac{ru'}{Rv_0}, \quad v'_* = \frac{rv'}{Rv_0}, \quad \eta'_* = \frac{r^2\eta'}{Rv_0}.$$

Here v_0 is the initial value of $\overline{v}$ at the inner cylinder *before* $\Delta\omega = 0$: $v_0 = \overline{v}(R)$. Then dropping the asterisks we can write for the new perturbation variables

$$\frac{\partial \eta'}{\partial t} + u'\frac{d\overline{\eta}}{dy} + \overline{w}\frac{\partial \eta'}{\partial x} = 0, \tag{7}$$

$$\partial u'/\partial y + \partial v'/\partial x = 0, \tag{8}$$

with
$$\eta' = \frac{\partial v'}{\partial y} - \frac{\partial u'}{\partial x}, \quad \frac{d\overline{\eta}}{dy} = \frac{d^2\overline{w}}{dy^2} + 2\frac{d\overline{w}}{dy}.$$

The boundary conditions are

$$u'(0) = u'(\ln(\epsilon+1)^{\frac{1}{2}}) = 0. \tag{9}$$

These equations are identical with the equations for a small disturbance in a two-dimensional flow in Cartesian co-ordinates, if we work with the vorticity gradient, which is here a little bit more complicated. The mathematical theory can be derived along the same lines without essential modifications (Foote & Lin 1950).

Now employing (8) we define a stream function for the perturbed motion:

$$u' = -\partial\psi/\partial x, \quad v' = \partial\psi/\partial y. \tag{10}$$

We consider the form

$$\psi(x, y, t) = \phi(y) \exp\{i\nu(x - ct)\}, \tag{11}$$

with ν a real dimensionless wavenumber and c a dimensionless phase velocity, which may be complex: $c = c_r + ic_i$. If $c_i \neq 0$, the stream function contains a term exponential in time, i.e. the wave is amplified if $c_i > 0$ and damped if $c_i < 0$; the neutral wave is characterized by $c_i = 0$.

On substituting (10) and (11) into (7) we obtain an equation for $\phi(y)$:

$$\phi'' - \nu^2\phi + \frac{\overline{w}'' + 2\overline{w}'}{c - \overline{w}}\phi = 0 \tag{12}$$

(in this connexion the prime denotes differentiation with respect to y); the boundary conditions (9) take the form

$$\phi(0) = \phi(\ln(\epsilon+1)^{\frac{1}{2}}) = 0. \tag{13}$$

Equations (12) and (13) describe a well-known eigenvalue problem.

The basic current

To solve the above eigenvalue problem we have to prescribe the basic current $\overline{w}$ in the interval $0 \leqslant y \leqslant \ln(\epsilon+1)^{\frac{1}{2}}$. For $\overline{w}$ we take

$$\overline{w} = A + e^{-y}\{a_1 \sin[(\alpha^2-1)^{\frac{1}{2}}y] + a_2\cos[(\alpha^2-1)^{\frac{1}{2}}y]\}, \tag{14}$$

assuming that this form will yield an adequate approximation to the actual velocity distribution with special consideration of the asymmetric structure. Equation (14) contains two special parameters A and α and two coefficients a_1 and a_2. From the conditions $\overline{w}(0) = \overline{w}(\ln(\epsilon+1)^{\frac{1}{2}}) = 0$, a_1 and a_2 can be determined:

$$a_1 = A\left[\frac{\cos((\alpha^2-1)^{\frac{1}{2}}\ln(\epsilon+1)^{\frac{1}{2}}) - (\epsilon+1)^{\frac{1}{2}}}{\sin((\alpha^2-1)^{\frac{1}{2}}\ln(\epsilon+1)^{\frac{1}{2}})}\right] = AB, \quad a_2 = -A. \tag{15}$$

In order to guarantee $\overline{w}_{\max} = 1$ the parameter A, an arbitrary dimensionless velocity, may be defined by

$$A = 1/F_{\max}, \tag{16}$$

where $$F_{\max} = \max\{1 + e^{-y}[B\sin((\alpha^2-1)^{\frac{1}{2}}y) - \cos((\alpha^2-1)^{\frac{1}{2}}y)]\},$$
$$0 \leqslant y \leqslant \ln(\epsilon+1)^{\frac{1}{2}}.$$

The quantity α is a characteristic parameter of the velocity profile $\overline{w}$ which reflects the structure of the observed profile (asymmetry and rate of decrease) and is based upon the dependence on ϵ.

Therefore the following relation for α seems to be reasonable:

$$\alpha = [(\tfrac{5}{4}\pi/\ln(\epsilon+1)^{\frac{1}{2}})^2 + 1]^{\frac{1}{2}}, \quad \epsilon > 0, \quad \text{i.e. in practice } \epsilon > 0{\cdot}2. \tag{17}$$

This relation guarantees that for increasing ϵ the asymmetry of the velocity profile increases and its rate of decrease diminishes at a realistic rate (see figure 11). In addition, the form of α has the effect of simplifying (15) in the right way:

$$a_1 = A\{1 + [2(\epsilon+1)]^{\frac{1}{2}}\}, \quad a_2 = -A. \tag{15a}$$

With (15a), (16) and (17) we obtain from (14)

$$\overline{w} = (F_{\max})^{-1}\{1 + e^{-y}[\{1 + (2(\epsilon+1))^{\frac{1}{2}}\}\sin\delta y - \cos\delta y]\}, \tag{18}$$
$$\delta = (\alpha^2-1)^{\frac{1}{2}} = \tfrac{5}{4}\pi/\ln(\epsilon+1)^{\frac{1}{2}}, \quad 0 \leqslant y \leqslant \ln(\epsilon+1)^{\frac{1}{2}}.$$

Figure 11 shows three velocity profiles according to (18) for $\epsilon = 0{\cdot}66$, $3{\cdot}0$ and $16{\cdot}0$. As a measure for asymmetry we define

$$\Delta_S = y_{\max}/y_b = y_{\max}/\ln(\epsilon+1)^{\frac{1}{2}}.$$

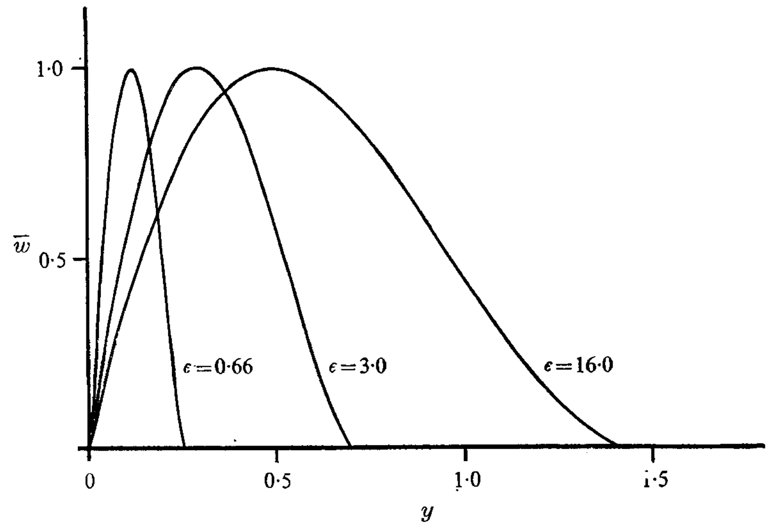

FIGURE 11. Basic currents $\overline{w}(y)$ for $\epsilon = 0{\cdot}66$, $3{\cdot}0$ and $16{\cdot}0$.

$\Delta_S = 0{\cdot}5$ then denotes symmetry. Calculating Δ_S for the three profiles in figure 11 we find a 6 % deviation from symmetry for $\epsilon = 0{\cdot}66$, 16 % for $\epsilon = 3{\cdot}0$ and 30 % for $\epsilon = 16{\cdot}0$.

Using (18) the vorticity gradient $d\overline{\eta}/dy$ takes the form

$$\frac{d\overline{\eta}}{dy} = \frac{d^2\overline{w}}{dy^2} + 2\frac{d\overline{w}}{dy} = \alpha^2(A - \overline{w}). \tag{19}$$

Introducing (19) into (12) we obtain

$$\phi'' - \nu^2\phi + \alpha^2\frac{A - \overline{w}}{c - \overline{w}}\phi = 0. \tag{20}$$

The neutral wave

Now, looking for the neutral solution, $c = c_r$, $c_i = 0$, the proper aim of the theoretical consideration, we proceed on the criterion that, if a neutral disturbance is to exist, $d\overline{\eta}/dy$ must vanish at all points y_c where $\overline{w} = c$.

Relation (19) then yields

$$d\overline{\eta}/dy = \alpha^2(A - \overline{w}) = 0,$$
$$\overline{w}(y_c) = A = c > 0.$$

c is the real phase velocity (dimensionless) of the neutral wave satisfying the condition

$$\overline{w}_{\min} < c < \overline{w}_{\max}.$$

A is defined by (16). From (20) it follows that the equation for the neutral wave is

$$\phi'' + \lambda^2\phi = 0 \quad (\lambda^2 = \alpha^2 - \nu^2), \tag{21}$$

with the boundary conditions (13). This eigenvalue problem has the solution

$$\phi = K\sin\lambda y, \quad \lambda = m\pi/\ln(\epsilon+1)^{\frac{1}{2}} \tag{22}$$

K an arbitrary constant, $m = 1, 2, \ldots.$

If we take the first eigenvalue and substitute relation (17) for α, equation (22) yields the real wavenumber ν as a function of ϵ for the neutral wave:

$$\nu = (\alpha^2 - \lambda^2)^{\frac{1}{2}} = \{[\tfrac{3}{4}\pi/\ln(\epsilon - 1)^{\frac{1}{2}}]^2 + 1\}^{\frac{1}{2}}. \tag{23}$$

The solid curve in figure 9 represents relation (23).

Conclusion

By applying Lin's method of calculation (1953, 1966) it can now be proved on the basis of relation (23) that in our case all waves with an integer wavenumber n smaller than the neutral ν (for the same ϵ) should amplify, whereas waves with a wavenumber n greater than this ν will be damped. A comparison with the experimental results (see figure 9) confirms the theoretical considerations. All unstable amplifying waves occurring in the experiments have an integer wavenumber n which is smaller than the corresponding ν; in almost all cases the n which is the nearest integer to ν, i.e. that corresponding exactly the first wavelength, which should amplify according to theory and corresponds to the shortest of the possible wavelengths, is selected from the spectrum.

As in our experiments, the basic currents described by the analytical expression (18) are unstable, because the condition for instability, that $d\bar{\eta}/dy$ must vanish somewhere within the y interval, is satisfied. So all things considered the special choice of $\bar{w}$ may be a sufficiently realistic approximation of the actual profile.

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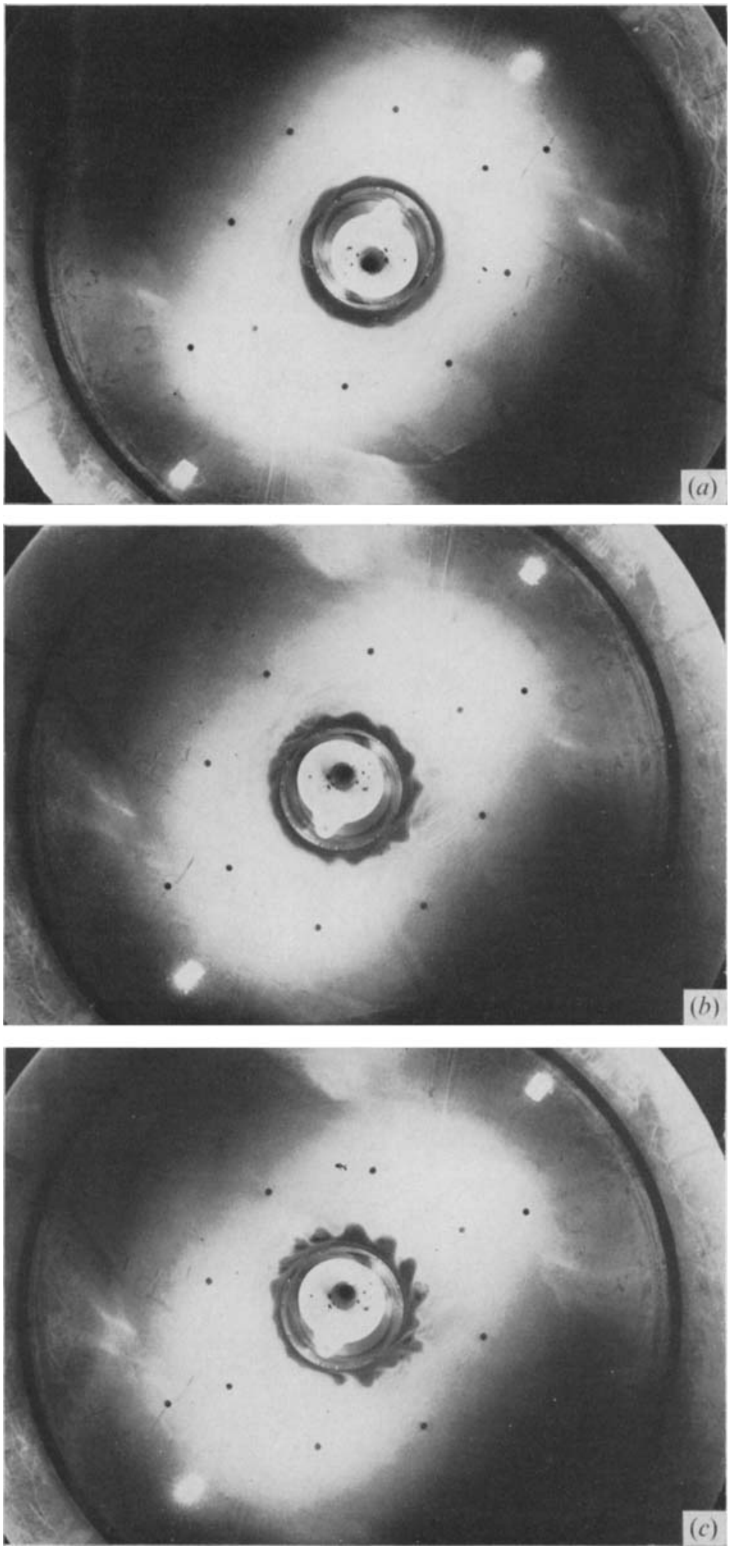

FIGURE 2. Experiment 1. Initial state $\epsilon = 0{\cdot}33$. Time after setting $\Delta\omega = 0$: (*a*) 50 s, (*b*) 80 s, (*c*) 110 s.

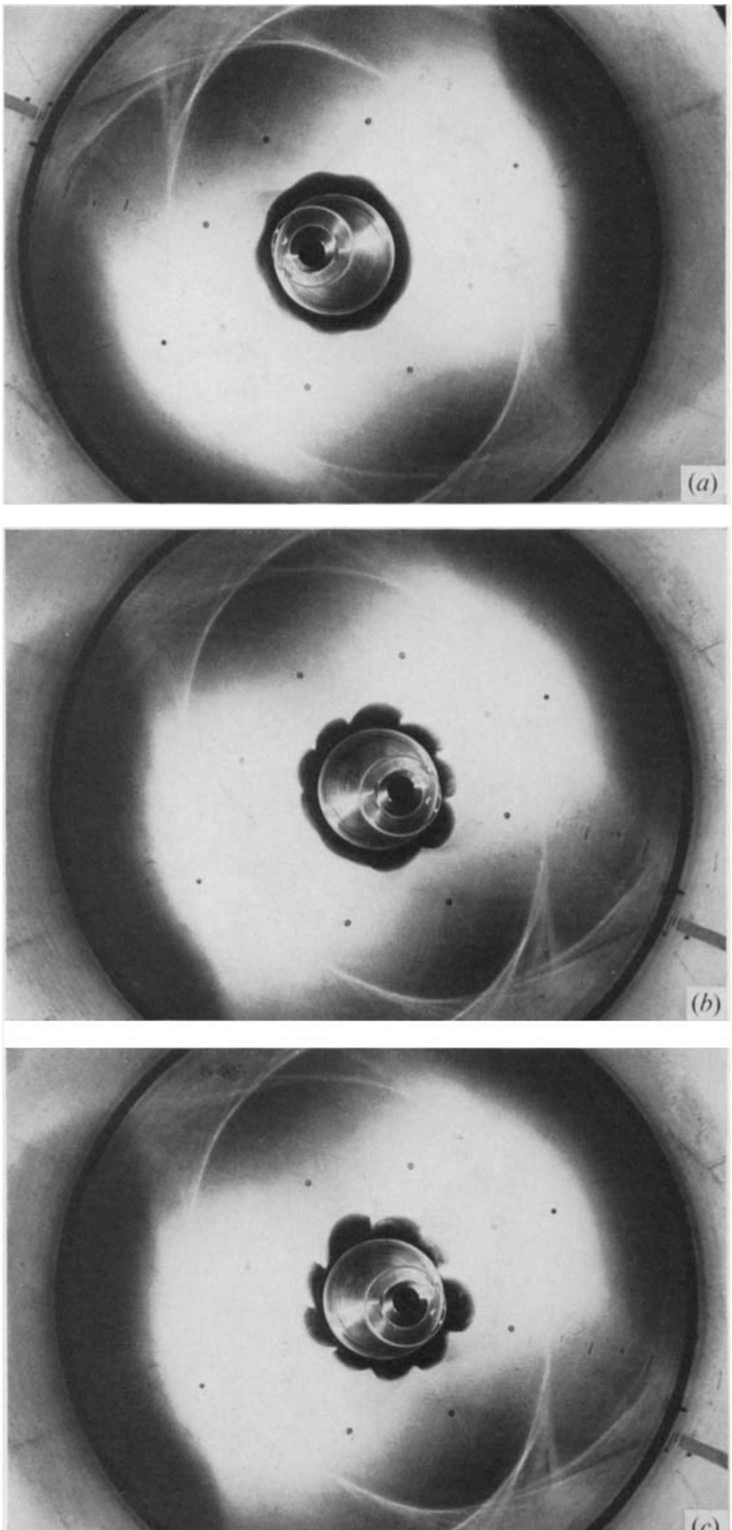

FIGURE 3. Experiment 2. Initial state $\epsilon = 0{\cdot}66$. Time after setting $\Delta\omega = 0$: (*a*) 15 s, (*b*) 30 s, (*c*) 45 s.

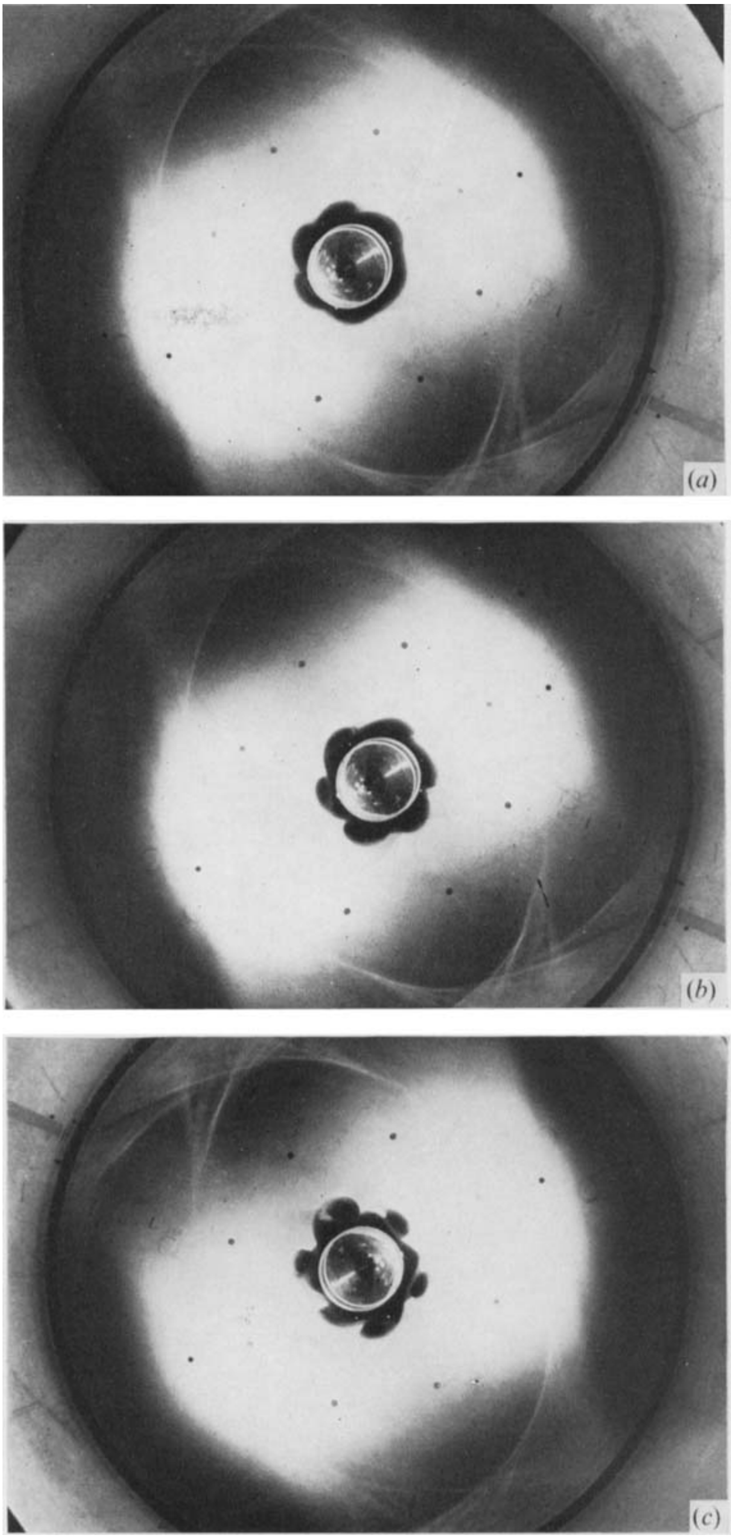

FIGURE 4. Experiment 3. Initial state $\epsilon = 1{\cdot}0$. Time after setting $\Delta\omega = 0$: (*a*) 20 s, (*b*) 35 s, (*c*) 55 s.

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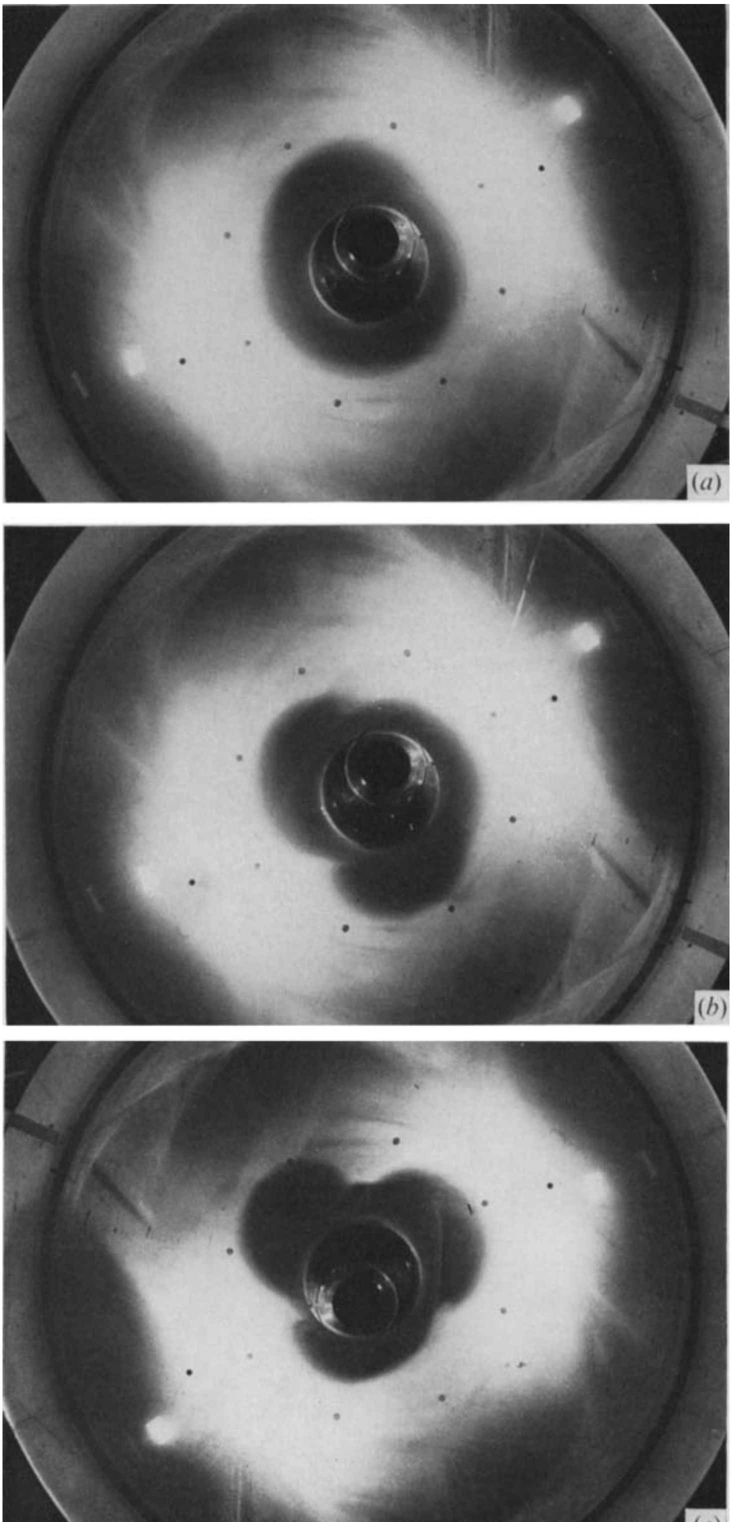

FIGURE 5. Experiment 4. Initial state $\epsilon = 3{\cdot}0$. Time after setting $\Delta\omega = 0$: (a) 15 s, (b) 20 s, (c) 25 s.

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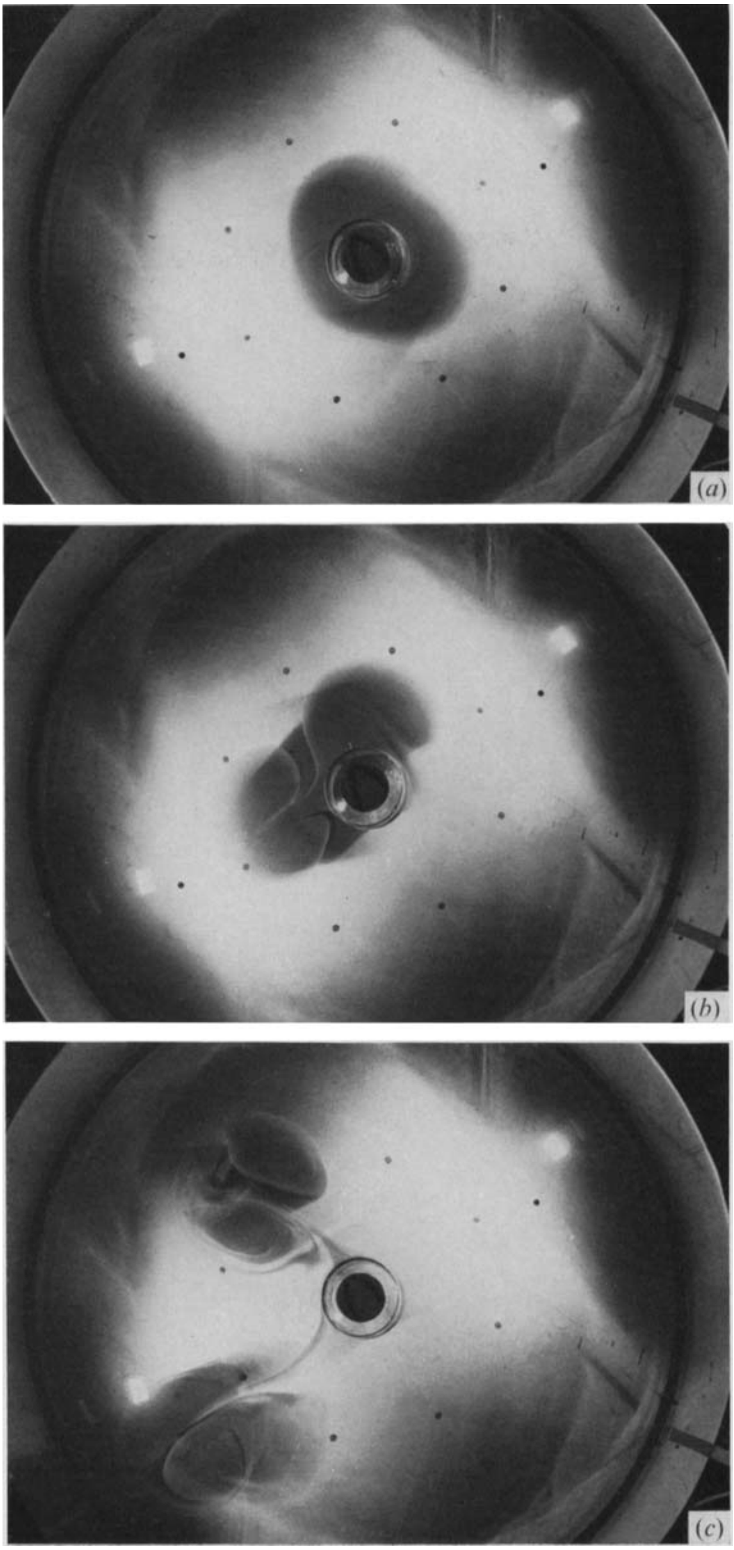

FIGURE 6. Experiment 5. Initial state $\epsilon = 5{\cdot}0$. Time after setting $\Delta\omega = 0$: (*a*) 25 s, (*b*) 35 s, (*c*) 50 s.

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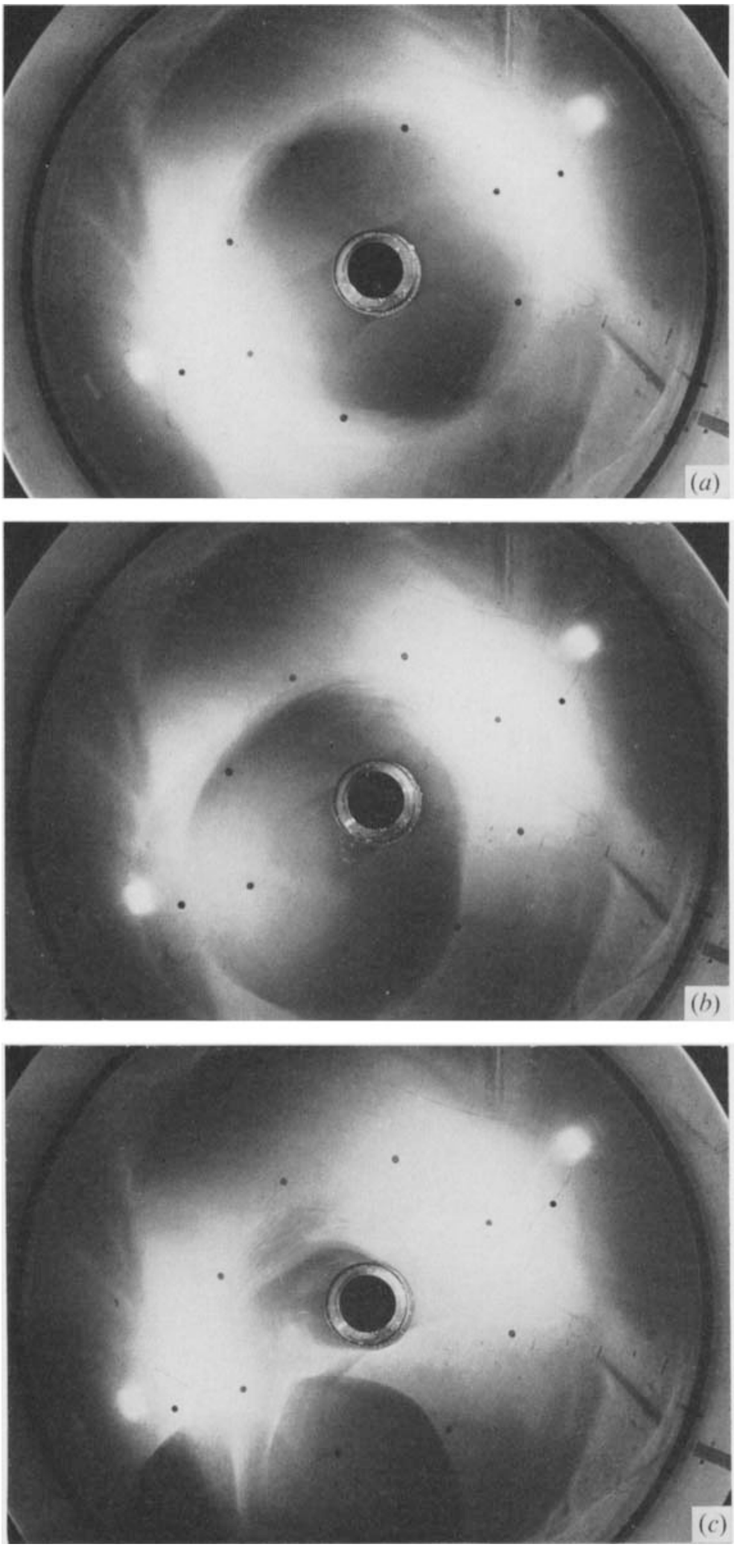

FIGURE 7. Experiment 6. Initial state $\epsilon = 16{\cdot}0$. Time after setting $\Delta\omega = 0$: (a) 15 s, (b) 40 s, (c) 55 s.

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